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ABSTRACT

Commonly used accepted procedures in the industry for calculating high collapse ratings of OCTG casing and tubing are described in Annexes G and H of API 5C3. Both approaches are limited to the analysis of historical data. Moreover, modern pipe dimensional mapping during production allows to provide properties specific to each individual pipe joint that were impossible before. In this paper, a method for calculating the collapse rating of each individual pipe produced is presented. In the developed method, the uncertainties on the quantification of each parameter are accounted for and the collapse rating of individual pipes is calculated to a pre-defined target reliability level (TRL). The statistical approach used to arrive at the collapse rating for a pipe joint is explained, and the consistency with the conventional method based on sample data is demonstrated. The developed method, therefore, enables practices long expected by the Oil & Gas industry, such as the clustering of the produced pipes based on their collapse performance. This allows the pipe manufacturer to assign high-performing pipes to the location of most severe loads in a well, while avoiding the delivery to the rig of the worst performing pipes.

INTRODUCTION

One of the foundations of the ongoing fourth industrial revolution, Industry 4.0, is the digital transformation, which potentially enables companies to increase process efficiency, improve data transparency and boost competitiveness (1). Digital Twins are at the core of the entire Industry 4.0 development, as they provide a near real-time bridge between the physical and digital worlds. According to the vision of Boschert et. al. (2), the vision of the Digital Twin refers to a comprehensive physical and functional description together with all available operational data of a component, product, or system, which includes more or less all information which could be useful in all lifecycle phases. Wright and Davidson (3) completed the concept, stating that digital twins can use any sort of model that is a sufficiently accurate representation of the physical object that is being twinned.

Modern mills producing Oil Country Tubular Goods (OCTG) used as casing and tubing in oil & gas wells have been offering solutions for advanced inspection of wall thickness and outer diameter (4; 5; 6; 7). When coupled with individual pipe physical traceability, these can provide detailed dimensional information for every segment of a pipe. To be able to use the dimensional data for individual performance calculation, the minimum requirement is to have a two-dimensional mapping of the pipe, i.e., the minimum, average and maximum values of both wall thickness and outer diameter for every segment of pipe. More advanced techniques may even yield a three-dimensional mapping of the pipe (8). This detailed information may unlock the creation of digital twin models of the first life cycles of the as-delivered pipes, conditioned to the existence of sufficiently accurate performance models, allowing operators to improve reliability, mitigate risks, and reduce operational expenses.

Capital expenses can also be reduced. For example, consider the scenario in which two products compete: product 1 is cheaper than product 2, but with lower performance (Figure 1). If the most stringent well conditions require as minimum acceptable rating that of product 2, the current optimized sourcing strategy should be to buy both products and place in different regions of the well. However, if the performance of each manufactured pipe is known, product 1 alone could be sourced and the individual pipes with performance higher than the minimum of product 2 can be segregated. This segregation approach was developed by Vallourec for high collapse offers under the name of Dual Performance Baskets (DPB) (9).

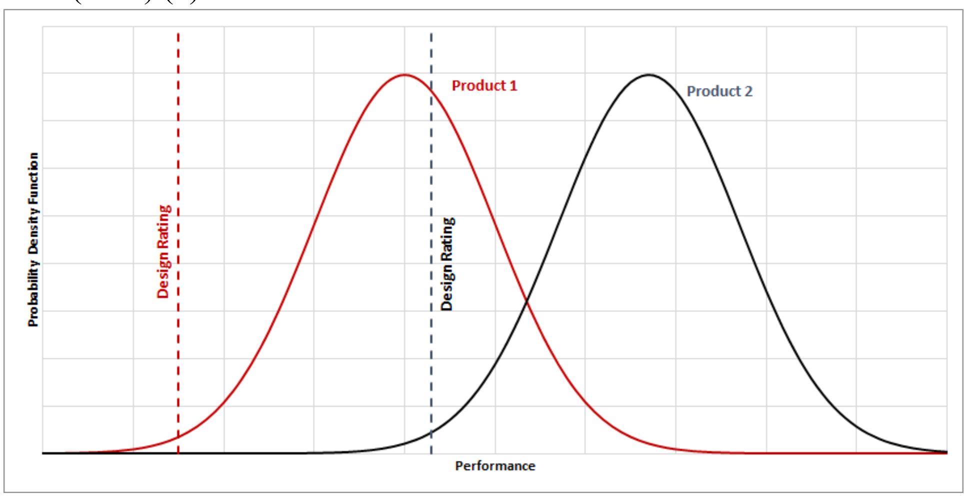

Figure 1 – Schematic representation the performance variance of two products

At first, we can use a straightforward definition of the performance rating for an individual pipe, which is based on deterministic approaches usually encoding working stress design, or allowable stress design (10). The dimensional mapping tied with the minimum mechanical properties defined by the applicable standards or custom manufacturer offer allows directly calculating the maximum effort applied to reach the allowable stress, usually chosen such that the material remains within its elastic limit. For example, for the calculation of the internal pressure to achieve the yield stress for a pipe with zero axial load, API 5C3 (11) defines:

$$p_{iYLo} = f_{ymn} \frac{\left(D^2 - d_{wall}^2\right)}{\sqrt{3D^4 + d_{wall}^4}} \qquad 1$$

where

D	is the specified pipe outside diameter;
d_{wall}	is the inside diameter based on $k_{wall}t$, $d_{wall} = D - 2k_{wall}t$;
f_{ymn}	is the specified minimum yield strength;
k_{wall}	is the factor to account for the specified manufacturing tolerance of the pipe wall;
p_{iYLo}	is the internal pressure at yield for an open-ended thick tube;
t	is the specified pipe wall thickness.

When dimensional mapping is available, t and D can be replaced respectively by the actual wall thickness and outer diameter of the pipe segment and k_{wall} can be set to the unity, leading to a direct calculation of the internal pressure at yield. A discussion remains on the possibility to adapt the safety factor used after the determination of p_{iYLo}, given that relevant information on the strength side is included during the design phase. However, this discussion pertains to each operator and is therefore outside the scope of this work.

On the other hand, if a probabilistic approach or reliability-based design is at stake, the calculation of the rating of an individual pipe is not as straightforward.

Typically, an acceptable probability of failure or exceedance (12) – or target reliability level (TRL) – is used to quantify the risk associated to a given resistance in a load scenario (13; 12; 14; 15). So, how should an acceptable TRL for a length of pipe be derived from the TRL value?

Secondly, only the dimensions are known for every segment of pipe, while other variables (e.g., mechanical properties) are not known for every segment of pipe. Thus, how should the unknown impacting variables be accounted for when calculating the resistance of an individual pipe?

To the best of the authors' knowledge, these questions have not been discussed in the literature. In this paper, we present an original approach to answer the questions above and risk-informed estimate the rating of an individual pipe. Addressing these questions is fundamental to properly develop Digital Twins.

COLLAPSE RESISTANCE OF OCTG CASING AND TUBING

Collapse resistance of pipe is a major factor in casing design for oil and gas wells. The pipe used as casing must be capable of withstanding any external pressure that occurs during drilling, completion, and production of the well. If it does not have adequate collapse resistance, the external pressure will cause it to collapse or flatten (16).

Collapse of pipe is an instability type of failure, analogous to buckling in columns. The collapse resistance can be affected by many factors, including the material Young's modulus, yield strength, and stress-strain curve shape; residual stress, diameter-to-wall thickness ratio, ovality, eccentricity, internal pressure, axial loading, bending and torsional stress. To accurately predict the ultimate collapse pressure $\hat{p}$ of a pipe, one relies on models including all those factors, derived from extensive testing campaigns. The performance of an ultimate limit state (ULS) model is given by the model uncertainty factor μ, a stochastic variable defined by the ratio between the actual collapse test pressure Pt and model prediction $\hat{p}$ (11):

$$\mu = \frac{Pt}{\hat{p}} \qquad (2)$$

Reliability analyses are then performed using the predictive ULS model multiplied by the model uncertainty factor. More details are presented in the Appendix.

The design rating is the one the operators can use as basis to perform well design, which may be reduced by a safety factor. Therefore, all segments of all pipes delivered by a manufacturer are very likely to remain above the given rating. For OCTG casing and tubing, historically there are four different ways of calculating the collapse design rating of a product described in API 5C3.

The historical API collapse equations, developed in the 1960s, give a lower bound for the design rating, partitioning the diameter-to-thickness domain into four regions: yield, plastic, transition, and elastic collapse. Except for the yield collapse model, which is a working stress design equation (16), all equations were aiming to represent the 0.5% TRL for the collapse sample (17; 18). The collapse database used was compound by 2488 tests in K55, N80 and P110 material. A discussion about the limitation of the historical approach can be found in Adams *et. al.* (19). Two main limitations are noteworthy: (1) as emerged in the 2000s, the risk level significantly varies across the diameter-to-thickness domain; (2) they apply to API products only and therefore are not applicable to proprietary High Collapse grades.

To address the first issue, the indirect approach was introduced in Annex F of API 5C3, using the Klever-Tamano ultimate limit state model (20) and the distributions of input variables drawn from a large sample collapse database. The resulting design model assures the 0.5% TRL is found throughout the diameter-to-thickness domain.

To address the second issue, API guidelines in Annex H of API 5C were issued to extend the application of the indirect method to a custom database. In this case, the Klever-Tamano ultimate limit state model is suggested to be used, but any other limit state model can be used, provided the model uncertainty is quantified by means of extensive collapse testing. The distributions of input variables should be drawn from the proprietary data. More details are presented in the Appendix. The 0.5% TRL is suggested to be followed.

Finally, as an alternative to the indirect method, the direct method was introduced in the Annex G of API 5C3. In the method, the collapse test results are used directly (i.e., without using a predictive model) to infer the design rating using a non-central student-t distribution. A custom database can be used to calculate the design rating. More details are presented in the Appendix. Once again, the 0.5% TRL is suggested to be used.

COLLAPSE RATING OF AN INDIVIDUAL PIPE

The TRL is defined at the collapse test sample level in API 5C3 (11), given the actual test result is used to yield the probability of failure in the direct method (Annex G) and the sample geometry measured according to the guidelines of Annex I is used in the indirect method (Annex H). How can we use that information to infer the design strength of an individual length of pipe?

An individual pipe may be viewed as a series system, composed of multiple segments as long as a collapse test sample – see Figure 2. In a series system, like in a chain, the failure of a single link results in the failure of the whole structure. Therefore, the probability of failure of the system (pipe) is function of the probabilities of failure of each and every link (segment).

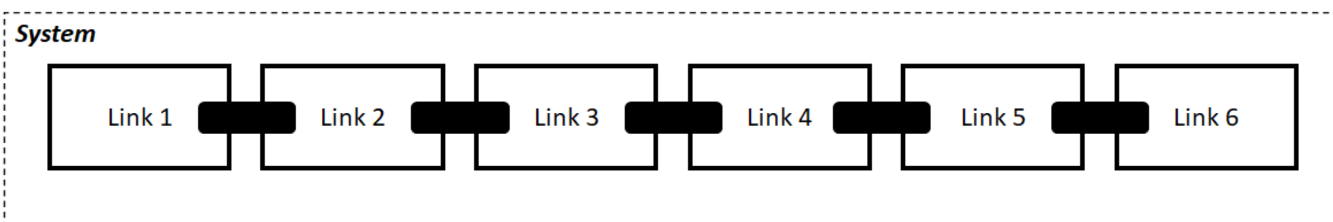

Figure 2 – Series system

If we assume that all links are independent, the upper bound for the probability of failure of the system is given by (21):

$$P_f = 1 - \prod_{i=1}^{n}\left(1 - P_{f,i}\right) \quad (3)$$

where $P_{f,i}$ if the probability of failure of the i-th link.

To derive the acceptable TRL for a pipe, TRL_p, from the TRL of a sample, TRL_s, it is realistic to assume all samples are independent. This is the basic assumption in the direct approach of API 5C3, and a reasonable approximation in the indirect approach.

Then, the TRL_p can be derived of a n collapse sample pipe reads:

$$TRL_p = 1 - \left(1 - TRL_s\right)^n \qquad 4$$

where all collapse samples, taken at random, have the same failure probability TRL_s. Or phrased differently: if we define the strength of the pipe as the minimum strength of the n samples within that pipe, this equation provides the probability of failure of the pipe as a function of the probability of failure of the sample.

The effect of n in TRL_p for a given TRL_s is illustrated in Figure 3, where we plot the distributions of the rating of a pipe made up of n samples, arranged in a series system, given an assumed normal distribution of a sample. For visualization, we do not report on the abscissa the actual rating scale.

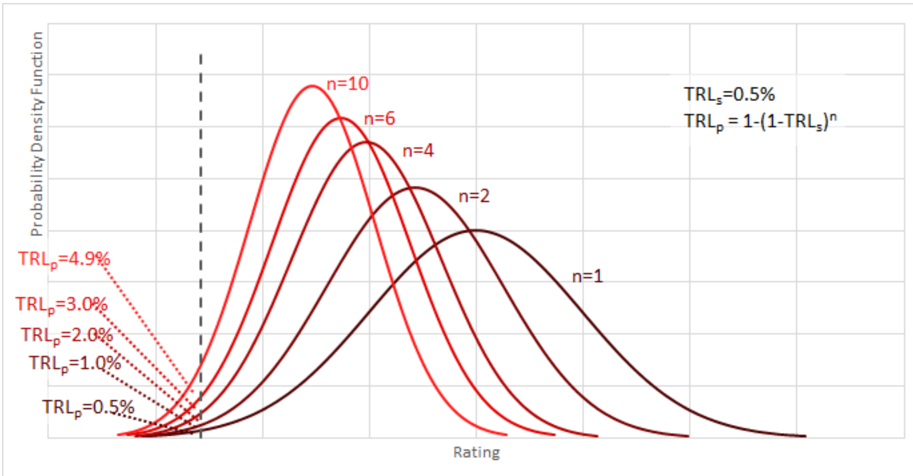

Figure 3 – TRL_p as function of n and TRL_s

We can see that the rating value corresponding to an unreliability value of 0.005 for the single sample corresponds to larger unreliability values for pipes composed of more samples: the larger the number of samples, the larger the probability of picking a weak sample at random. Therefore, to assure equivalence between the API 5C3 approaches and the individual pipe analysis approach, the acceptable TRL_p increases with increasing n as per Equation 4, because, after all, we take the pipe strength as the minimum over the n samples within that pipe.

On the other hand, we must consider that the statistical independence of the samples is a very strong assumption. To see this, Figure 4 and Figure 5 show the normalized values of the collapse rating for 20 pipes of 9 samples each, sampled from a normal distribution $N(1,0.1)$, considering the two extreme cases in which the samples are statistically independent (Figure 4) and the case in which they are fully correlated (i.e., correlation coefficient=1, Figure 5). In the former case, we observe that the collapse pressure can vary up to around 30% from one link to its neighbors. This corresponds to the situation in which the production settings at the mill can abruptly vary along the production of the same pipe. In the latter case, we can observe that the collapse pressure only slightly oscillates around the value of the first sample. This corresponds to the situation in which the production settings can be considered stationary, with some variability due to small noisy factors.

Of course, the case of complete correlation among the pipe samples may be dangerously non conservative.

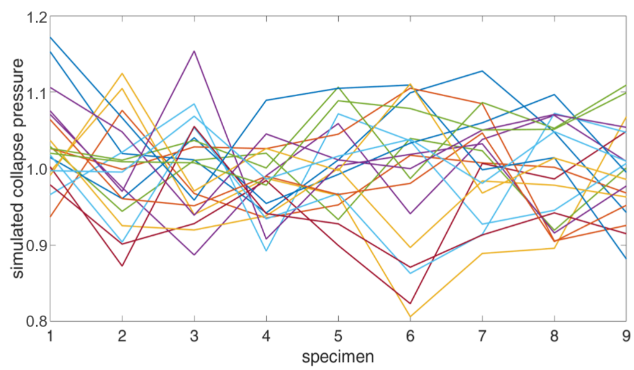

Figure 4 – Rating values for independent samples

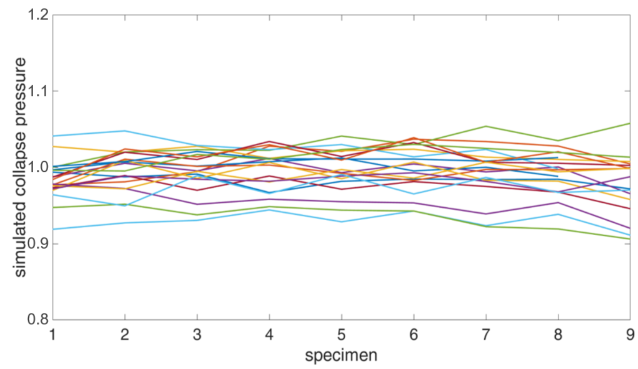

Figure 5 – Rating values for fully correlated samples

The actual probability of failure of the pipe system is likely to be between these two extreme situations. For example, Figure 6 shows the variability over the 9 samples of 20 pipes considering correlation coefficient decreases with the distance of the samples. This represents the situation in which consecutive samples are likely to have similar properties. We can then realize that the variability of the collapse pressure values along the pipe corresponding to this modeling assumption about correlation is in between those of Figure 4 and Figure 5.

The probability of failure of the pipe can be found via Monte Carlo simulation, as shown in the Appendix.

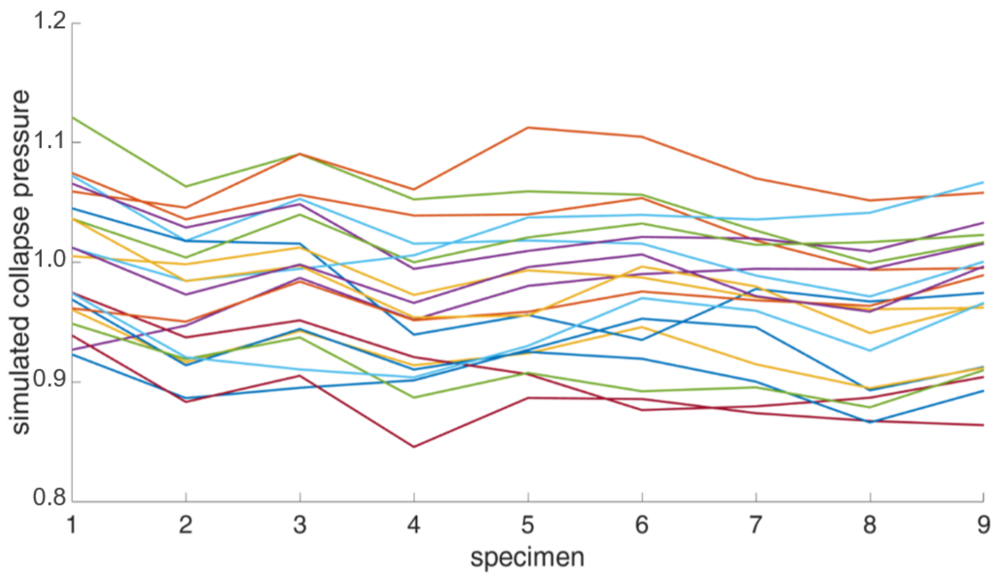

Figure 6 – Rating values for correlation decreasing with the distance among the specimens

RESULTS

The method was applied to multiple productions of a 16.08” outer diameter 109.9lb/ft (0.667” wall thickness) VM 140 CYHC, with nominal collapse rating of 5,130psi. The VM 140 CYHC is a Vallourec proprietary grade with yield strength ranging between 140ksi and 150ksi. One example of the dimensional mapping data available for each produced pipe is presented in Figure 7.

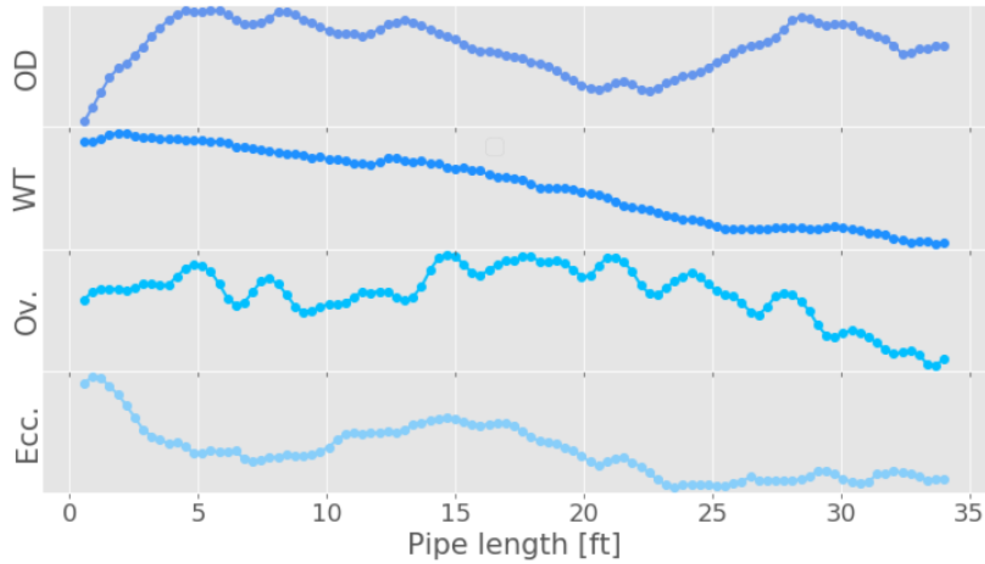

Figure 7 – Dimensional mapping of one pipe

The hyperparameters and correlation of the mechanical properties, residual stress, and model uncertainty distributions were calculated based on long-lasting mill experience with the product, backed up by thousands of physical tests, and on in-lot measurements. A proprietary collapse model was used for the calculation of the sample ratings and the method described above to calculate the individual pipe ratings.

Figure 8 shows the difference between the reliability of 100 joints assuming that the specimens are independent on one another (blue), and assuming two different conditions for the correlation decreasing with distance (red and green). We can observe that differently from the case of independent specimens, it is not possible to encounter situations in which some specimens have a collapse pressure close to each other while others are far.

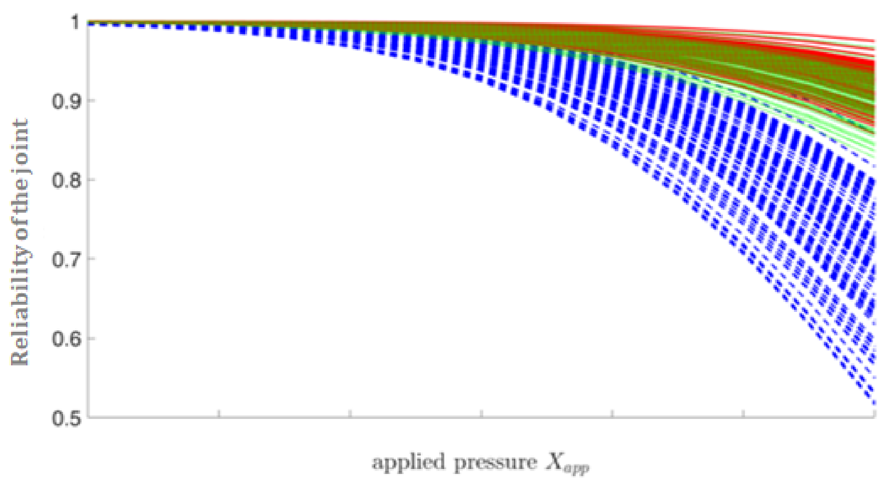

Figure 8 Reliability vs applied pressure for the all the joints, considering constant correlation (red), linearly decreasing correlation (green) and assuming independence between specimens (blue).

For 24 of the pipes produced, a collapse test sample was randomly taken along the pipe axis. The margin of the actual (tested) collapse pressure of each sample to the calculated individual rating of the mother pipe was plotted in Figure 9. One might observe that minimum margin safely lays 3% above the calculated individual pipe rating.

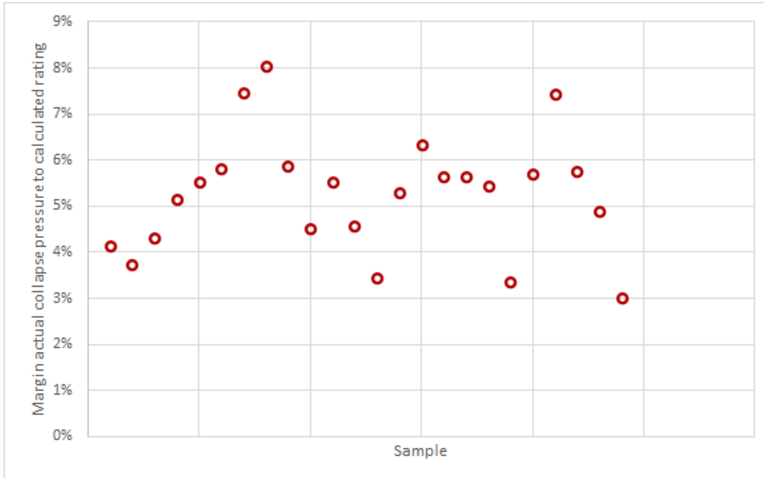

Figure 9 – Margin of actual collapse pressure to calculated pipe rating

CONCLUSIONS

Modern pipe dimensional mapping makes it possible for a pipe manufacturer to deliver a strength rating to each individual joint of pipe. In this paper, an original method is provided on how to calculate the pipe joint collapse strength.

The pipe joint strength is defined as the minimum strength of the samples within that joint. The joint rating as calculated from the joint strength distribution assures equivalence to the API 5C3 approaches based on sample strength distribution.

In summary, the method describes how to:

- account for the uncertainties and correlation of the variables that may impact the collapse performance
- define the acceptable TRL of a joint based on the TRL for the sample strength distribution

The expected benefit is that receiving a strength rating for each individual pipe joint allows the end user to position the higher strength joints at the locations in the well where the loads are highest. Such practice may be an enabler for challenging projects and/or allow the reduction of risks associated to collapse loads.

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APPENDIX

Approach by Annex G of API 5C3

The pipe pressure design value of p_{des} depends on the link TRL, here indicated by r. This is a design requirement indicating the portion of links that are expected to withstand pressure values larger than p_{des}. Typically, r =0.995.

We indicate by X the actual collapse pressure of a generic sample. This is a random variable linked to r and p_{des} through the following Equation:

$$r = P(X \geq p_{des}) \qquad \text{A. 1}$$

Notice that Eq. A. 1 encodes two sources of uncertainty:

- The uncertainty in the actual sample collapse value X, which represents the inherent variability within a homogeneous population of samples (e.g., due to the manufacturing process, materials, etc.). If we exactly knew the distribution of X, then we could estimate the value of p_{des} corresponding to r. In this setting, p_{des} is the $1 - r^{th}$ percentile of the collapse. To highlight that p_{des} depends on r, we use notation $p_{des}(r)$.
- The uncertainty in the estimation of X. We do not know exactly the distribution of the actual collapse of the sample, as we infer it from a limited number of samples N_{test}. Performing the procedure described in Annex G of API 5C3 on different batches of collapse test values gives different estimations of the empirical distribution of X and, thus, different values of $p_{des}(r)$. To take this into account, we indicate by $\hat{p}_{des}(r, N_{test})$ the estimation of $p_{des}(r)$ corresponding to a single batch of N_{test} samples and TRL set to r. Given the variability of $\hat{p}_{des}(r, N_{test})$, we consider the one-sided $(1 - \alpha)\%$ confidence interval on this estimator and, to be conservative, we consider as pipe collapse strength the lower bound of this interval $\hat{p}_{des}^{1-\alpha}(r, N_{test})$ such that

$$1 - \alpha = P(\hat{p}_{des}^{1-\alpha}(r, N_{test}) \leq p_{des}(r)) \qquad \text{A. 2}$$

In words, $\hat{p}_{des}^{1-\alpha}(r, N_{test})$ is such that $(1 - \alpha)\%$ of times the estimation from a random N_{test} pipe batch is smaller than its actual value $p_{des}(r)$ (i.e. the value that would be obtained if the probability distribution of X was exactly known). This entails that with probability $(1 - \alpha)$ the procedure is conservative and underestimates the value of the collapse pressure. Of course, the larger the value of N_{test}, the smaller is the uncertainty in the estimations of $\hat{p}_{des}^{1-\alpha}(r, N_{test})$, which approaches p_{des}.

Approach considering the geometrical parameters

Consider a model for pressure collapse estimation, which encodes geometrical and mechanical parameters, here indicated by θ.

$$X = \mu \cdot \hat{p}(\theta) \qquad \text{A. 3}$$

If the distributions of the uncertain parameters θ and μ were perfectly known, then the design collapse pressure $p_{des,s}(r)$ of specimen s would be the $1 - r^{th}$ percentile of the distribution of X_s derived by propagating both the uncertainty on θ through model $\hat{p}$ and the model uncertainty μ.

However, this is not so: we consider the epistemic uncertainty in the hyper-parameters of the distributions of both μ and θ.

To build a confidence interval for the design collapse pressure, we can apply the following Monte Carlo procedure for specimen s:

For $i = 1, ..., N_{Param}$:

- For each hyper-parameter, sample a value from its (usually normal) distribution.
- Inner Monte Carlo loop with N_{MC} samples to propagate the uncertainty through collapse model $\hat{p}$ and get the distribution X^{i} :
- For $\rho = 1, ..., N_{MC} \gg 1$, do:
    - Sample θ_{ρ} and U_{ρ}
    - Compute $X_{s}^{i}(\rho)$
- Take the $1 - r^{th}$ percentile $\hat{p}_{des,s}^{i}(r)$ of $X_{,s}^{i}(1), ..., X_{s}^{i}(N_{MC})$

Then, the $1 - \alpha^{th}$ percentile of the gathered values $\hat{p}_{des,s}^{i}(r), i \in \{1, ..., N_{param}\}$ is the lower bound of the confidence interval for the estimation of the $1 - r^{th}$ percentile of the collapse pressure. This is the design pressure value $\hat{p}_{des,s}(r)$.

From specimen to pipe

Consider a pipe partitioned in N_{s} specimens. Its reliability, $RJ\left(p_{des}\right)$, for a specific external pressure p_{des}, depends on that of the specimens, whose collapse pressure values $X = \left[X_{1}, ..., X_{N_{s}}\right]$ are random variables (estimated as explained before).

$RJ\left(p_{des}\right)$ is the reliability of a series system, which depends on the multivariate distribution indicating the probability that the collapse pressure of all specimens is above p_{des}:

$$RJ = P\left(\left\{X_{1}, ..., X_{N_{s}}\right\} \geq p_{des}\right) \qquad \text{A. 4}$$

The epistemic uncertainty on the reliability of a specific specimen propagates onto the joint reliability, whereby we indicate by $\hat{RJ}(p_{des})$ the estimator of the reliability of the joint for external pressure p_{des}.

We first develop the procedure to estimate $\hat{RJ}(p_{des})$ and the related confidence interval, under the assumption that all the collapse pressure values $X_{1}, ..., X_{N_{s}}$ in the same joint are statistically independent.

For every specimen $s = 1, ..., N_{s}$, repeat for $i = 1, ..., N_{Param} \gg 1$ the following procedure:

- For each hyper-parameter of both μ and θ, sample a value from the corresponding distribution.
- Compute $\hat{R}_{s}(i)$, the reliability of the specimen for the applied pressure p_{des} with the sampled hyper-parameters. This value is obtained via an inner Monte Carlo loop:
- For $\rho = 1, ..., N_{MC} \gg 1$, do:
    - Sample R and U_{ρ}

- Compute X_ρ using Eq. A. 3
- $B_\rho = 1$ if $X_\rho > p_{des}$, $B_\rho = 0$ otherwise

- $\hat{R}_s(i) = \frac{\sum_{\rho=1}^{N_{rel}} B_\rho}{N_{rel}}$

Then, we can compute for each trial $i = 1, ..., N_{Param}$, the reliability of the corresponding series system (i.e., the pipe) as $\hat{RJ}(i) = \prod_{s=1}^{N_{s,J}} \hat{R}_s(i)$. The distribution of $\hat{RJ}(i)$ represents the epistemic uncertainty in the value of the reliability of the series system due to the fact that the hyper-parameters are estimated from few samples. We take the $1 - \alpha$ percentile of this distribution to estimate $\hat{RJ}^{1-\alpha}$.

Correlation among the specimens of the joints

The collapse pressure in a joint is assumed stationary. Then, each parameter θ_j, j=1…,|θ| has a constant mean along the pipe.

With respect to the covariance matrix Σ_{θ_j}, we consider the correlation matrix Γ_{θ_j} such that $\Sigma_{\theta_j} = \sigma_{\theta_j}^T \cdot \Gamma_{\theta_j} \cdot \sigma_{\theta_j}$. The diagonal entries of Γ_{θ_j} are all set to 1, whereas the other entries $\gamma_{k,l}$, ranging in [-1,1], represent the correlation factor between the different specimens.

To simplify the notation, we omit to consider θ_j in $\gamma_{k,l}$ even if it can be different for each parameter considered.

We assume that the correlation between specimens depends only on their spatial distance τ. That is, $\gamma_{k,l} = \gamma(|k - l|) = \gamma(\tau)$, $\tau = |k - l|$, $\tau = 1, ..., N_{S,j} - 1$. Notice that if all the specimens are independent on each other, $\gamma(\tau) = 0\ \forall \tau > 1$. The values of the correlation parameters can be estimated through traditional statistical techniques.

Finally, we apply a procedure like that reported in the previous section: Repeat the following procedure for $i = 1, ..., N_{Param} \gg 1$

- Sample a value from for each unknown hyper-parameter, according to the corresponding distributions. For every hyper-parameter j, the mean $\mu_{\theta_j}(i)$ and standard deviation $\sigma_{\theta_j}^2(i)$ are the same for all the samples of the pipe.
- Compute $\hat{RJ}(i)$, the reliability of the specimen against the applied pressure p_{des} with the sampled hyper-parameters through the inner loop of Monte Carlo simulations:
- For $\rho = 1, ..., N_{MC}$ times, do the following procedure:
  - Sample $\theta_{j,\rho}$ and U_ρ from the corresponding multivariate distributions (i.e., considering correlation).
  - Compute $X_{\rho,s}$ using Eq. A. 3 for each specimen $s \in \{1, ..., N_{s,J}\}$.
  - Then, $X_\rho = min_{s=1,...N_s} X_{\rho,s}$, i.e., the collapse pressure is the minimum among all the specimens.
  - $B_\rho = 1$ if $X_\rho > X_{app}$, $B_\rho = 0$ otherwise

- $\hat{RJ}(i) = \frac{\sum_{\rho=1}^{N_{rel}} B_{\rho}}{N_{rel}}$

Then, we take the $1 - \alpha$ percentile of the distribution to give the estimate $\hat{RJ}^{1-\alpha}$, considering the correlation among the specimens in the same joint.